Advanced operations research Name: Time: 120 min

1. Show that if C is a convex cone, then C has at most one extreme point, namely, the origin.

2. Show that three definitions of extreme points are equivalent.

3. A necessary and sufficient condition for unboundedness of the objective value of a (feasible) minimization problem is that there exists a direction of the feasible region such that $cd < 0$. A condition for unboundedness in the simplex method is the existence of an index j such that $z_j - c_j > 0$ and $y_j \leq 0$. Discuss in detail the correspondence between the two conditions. (Hint: Let $d = [-y_j, 0, ..., 1, ..., 0]^t$,where the 1 appears at the jth position. Show that d is a direction of the set and that $cd = c_j - z_j$. Can you show that d is an extreme direction?)

4. Use the single artificial variable technique to solve the following linear programming problem:

$$\min -x_1 - 2x_2 + x_3$$
$$x_1 + 2x_2 + x_3 \geq 4$$
$$2x_1 - x_3 \geq 3$$
$$x_2 + x_3 \leq 2$$
$$x_1, x_2, x_3 \geq 0.$$

5. Show that lexicographic rule prevents from cycling.

6. Consider the following problem:

$$\min Z(x) = cx$$
$$Ax = b.$$

Show that if $Rank(A) = Rank(\frac{A}{c})$, then $Z(x) = cx$ for every x has a constant value